N19 - A0101

TED (15) – 1002

(REVISION — 2015)

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DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/ MANAGEMENT/COMMERCIAL PRACTICE — OCTOBER, 2019

ENGINEERING MATHEMATICS - I

[*Time* : 3 hours

(Maximum marks : 100)

PART — A

(Maximum marks : 10)

Marks

I   Answer *all* questions. Each question carries 2 marks.

1. Prove that $\cos^2 A - \sin^2 A = 2 \cos^2 A - 1$.

2. Write the expression for $\sin 3A$.

3. Prove that in any triangle ABC, $abc = 4R\Delta$.

4. If $y = x \sin x$, Find $\dfrac{dy}{dx}$

5. Find the velocity and acceleration at time 't' of a particle moving according to
   $s = 2t^3 - 3t^2 + 1$.   (5×2 = 10)

PART — B

(Maximum marks : 30)

II   Answer any *five* of the following questions.   Each question carries 6 marks.

1. Express $4 \cos x + 3 \sin x$ in the form $R \sin(x + \alpha)$ where α is acute.

2. Prove that $\sin 10° \sin 50° \sin 70° = \dfrac{1}{8}$.

3. Prove that $(a - b)\cos \dfrac{c}{2} = c \sin \dfrac{A-B}{2}$.

4. Differentiate $\sin x$ by the method of first principles.

5. Find $\dfrac{dy}{dx}$ if $(x^2 + y^2)^2 = xy$.

6. Find the equation to the tangent and normal to the curve $y = 3x^2 + x - 2$ at $(1, 2)$.

7. Prove that $\sin A + \sin(120° + A) + \sin(240° + A) = 0$.   (5×6 = 30)

[2]

PART — C

Marks

(Maximum marks : 60)

(Answer *one* full question from each unit. Each full question carries 15 marks.)

Unit — I

III (a) Prove that $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\,\text{cosec }\theta$.   5

(b) If θ is acute and $\sin\theta = 0.4$, find the value of $\sec\theta + \tan\theta$.   5

(c) If $A + B = 45°$, show that $(1 + \tan A)(1 + \tan B) = 2$.   5

Or

IV (a) Prove that $\dfrac{1+\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1-\cos\theta}$.   5

(b) If $\sin A = \dfrac{4}{5}$, $\sin B = \dfrac{12}{13}$; A, B are acute, find $\sin(A+B)$ and $\cos(A-B)$.   5

(c) The horizontal distance between two towers is 60m and the angle of depression of the first tower as seen from the second which is in 150m height is 30°. Find the height of the first tower.   5

Unit — II

V (a) Prove that $\dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A} = 2$.   5

(b) Prove that $\tan A + \cot A = 2\,\text{cosec }2A$.   5

(c) Show that $\dfrac{\sin 2A}{1+\cos 2A} = \tan A$ and deduce the value of $\tan 15°$.   5

Or

VI (a) Prove that $\dfrac{\sin A+\sin 3A+\sin 5A}{\cos A+\cos 3A+\cos 5A} = \tan 3A$ 5

(b) Prove that $\sin A + \sin 3A + \sin 5A + \sin 7A = 4\cos A\cos 2A\sin 4A$.   5

(c) Solve $\triangle ABC$, given $a = 4cm$, $b = 5cm$ and $c = 7cm$.   5

Unit — III

VII (a) Evaluate $\lim_{x\to 4} \dfrac{x^4-256}{x^3-64}$.   5

(b) If $x = a(\theta - \sin\theta)$; $y = a(1 - \cos\theta)$, show that $\dfrac{dy}{dx} = \cot\dfrac{\theta}{2}$ 5

(c) If $y = A\cos px + B\sin px$, (A, B, p are constants), Show that $\dfrac{d^2y}{dx^2}$ is proportional to y.   5

Or

Marks

VIII (a) Evaluate (i) $\lim_{x \to 0} \dfrac{1 - \cos 2x}{x^2}$ (ii) $\lim_{x \to -1} \dfrac{x^3 + 1}{x + 1}$ (3+3=6)

(b) Find $\dfrac{dy}{dx}$ if $y = (x^2 + x + 1)^7 \sin^2 x$. 4

(c) If $y = Ae^{nx} + Be^{-nx}$ (A, B are constants), Show that $\dfrac{d^2y}{dx^2} - n^2 y = 0$. 5

Unit — IV

IX (a) A particle is projected vertically upwards and its height 'h' and time 't' are connected by $h = 60t - t^2$. Find the greatest height attained. 5

(b) A balloon is spherical in shape. Gas is escaping from it at the rate of 10cc/sec. How fast is the surface area shrinking when the radius is 15cm. 5

(c) The deflection of a beam is $S = 2x^3 - 9x^2 + 12x$. Find the maximum deflection. 5

Or

X (a) Find the velocity and acceleration of a particle at t = 3 seconds whose displacement is given by $S = 3t^3 - t^2 + 9t + 1$. 5

(b) A spherical balloon is inflated by pumping 25cc of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15cm. 5

(c) Find the maximum value of $2x^3 - 3x^2 - 36x + 10$. 5