N19 - A0101

TED (15) – 1002 Reg. No...

(REVISION — 2015) Signature ....

DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/ MANAGEMENT/COMMERCIAL PRACTICE — OCTOBER, 2019

ENGINEERING MATHEMATICS - I

[*Time* : 3 hours

(Maximum marks : 100)

PART — A

(Maximum marks : 10)

Marks

I Answer *all* questions. Each question carries 2 marks.

1. Prove that $\cos^2 A - \sin^2 A = 2\cos^2 A - 1$.
2. Write the expression for sin 3A.
3. Prove that in any triangle ABC, $abc = 4R\Delta$.
4. If $y = x \sin x$, Find $\frac{dy}{dx}$
5. Find the velocity and acceleration at time *'t'* of a particle moving according to $s = 2t^3 - 3t^2 + 1$. (5×2 = 10)

PART — B

(Maximum marks : 30)

II Answer any *five* of the following questions. Each question carries 6 marks.

1. Express $4\cos x + 3\sin x$ in the form $R\sin(x + \alpha)$ where α is acute.
2. Prove that $\sin 10° \sin 50° \sin 70° = \frac{1}{8}$.
3. Prove that $(a - b)\cos\frac{C}{2} = c \sin\frac{A-B}{2}$.
4. Differentiate sin x by the method of first principles.
5. Find $\frac{dy}{dx}$ if $(x^2 + y^2)^2 = xy$.
6. Find the equation to the tangent and normal to the curve $y = 3x^2 + x - 2$ at (1, 2).
7. Prove that $\sin A + \sin(120° + A) + \sin(240° + A) = 0$. (5×6 = 30)

Marks

PART — C

(Maximum marks : 60)

(Answer *one* full question from each unit. Each full question carries 15 marks.)

UNIT — I

III (a) Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2 \operatorname{cosec} \theta$. 5

(b) If θ is acute and $\sin \theta = 0.4$, find the value of $\sec \theta + \tan \theta$. 5

(c) If $A + B = 45°$, show that $(1 + \tan A)(1 + \tan B) = 2$. 5

OR

IV (a) Prove that $\frac{1+\cos\theta}{\sin\theta} = \frac{\sin\theta}{1-\cos\theta}$. 5

(b) If $\sin A = \frac{4}{5}$, $\sin B = \frac{12}{13}$; A, B are acute, find $\sin (A + B)$ and $\cos (A - B)$. 5

(c) The horizontal distance between two towers is 60m and the angle of depression of the first tower as seen from the second which is in 150m height is 30°. Find the height of the first tower. 5

UNIT — II

V (a) Prove that $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$. 5

(b) Prove that $\tan A + \cot A = 2\operatorname{cosec} 2A$. 5

(c) Show that $\frac{\sin 2A}{1+\cos 2A} = \tan A$ and deduce the value of $\tan 15°$. 5

OR

VI (a) Prove that $\frac{\sin A+\sin 3A+\sin 5A}{\cos A+\cos 3A+\cos 5A} = \tan 3A$ 5

(b) Prove that $\sin A + \sin 3A + \sin 5A + \sin 7A = 4 \cos A \cos 2A \sin 4A$. 5

(c) Solve Δ ABC, given a = 4cm, b = 5cm and c = 7cm. 5

UNIT — III

VII (a) Evaluate $\lim_{x\to 4} \frac{x^4 - 256}{x^3 - 64}$. 5

(b) If $x = a(\theta - \sin \theta)$; $y = a(1 - \cos \theta)$, show that $\frac{dy}{dx} = \cot \frac{\theta}{2}$ 5

(c) If $y = A \cos px + B \sin px$, (A, B, p are constants), Show that $\frac{d^2y}{dx^2}$ is proportional to y. 5

OR

Marks

VIII (a) Evaluate (i) $\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$ (ii) $\lim_{x\to -1} \frac{x^3+1}{x+1}$ (3+3=6)

(b) Find $\frac{dy}{dx}$ if $y = (x^2 + x + 1)^7 \sin^2 x$. 4

(c) If $y = Ae^{nx} + Be^{-nx}$ (A, B are constants), Show that $\frac{d^2y}{dx^2} - n^2y = 0$. 5

UNIT — IV

IX (a) A particle is projected vertically upwards and its height '*h*' and time '*t*' are connected by $h = 60t - t^2$. Find the greatest height attained. 5

(b) A balloon is spherical in shape. Gas is escaping from it at the rate of 10cc/sec. How fast is the surface area shrinking when the radius is 15cm. 5

(c) The deflection of a beam is $S = 2x^3 - 9x^2 + 12x$. Find the maximum deflection. 5

OR

X (a) Find the velocity and acceleration of a particle at t = 3 seconds whose displacement is given by $S = 3t^3 - t^2 + 9t + 1$. 5

(b) A spherical balloon is inflated by pumping 25cc of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15cm. 5

(c) Find the maximum value of $2x^3 - 3x^2 - 36x + 10$. 5